Mechanism Design of South Pointing Chariots

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Mechanism Design of South Pointing Chariots

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South Pointing Chariots are based on a mechanical system using a differential gearing compensation between left and right wheels, so that the pointer atop a vertical axle of the chariot will always point in the same direction. It is a premium invention from ancient China.

1. Preface

The South Pointing Chariot (SPC) is a tool used to indicate direction. According to legend, Huang Di, the yellow emperor, designs the first SPC during "Zhu Lu Da Zang" (the Great War) in the primitive society. Presumably this is only a legend, but we don't know for sure. Another legend says that Chou Kung designed such a chariot. But from today's point of view it is hard to believe that those times could have had the workmanship to manufacture a South Pointing Chariot. But does the invention not signify the existence of the first civilization? In order to avoid a contention between the first and second civilization, written history has it that some Ma Chün constructs his first one for Emperor Ming Ti. Tsu Chhung-Chih and Chin Kung-Li successfully made improved chariots for emperor Shun Ti and Thang. But they do not leave any technical description.

Only during the reign of Emperor Jen Tsung the South Pointing Chariots designed by Yen Su and Wu Tê-Jen got their specifications about form and construction preserved in the historical record. The principle is similar to today's differential gearing.

In this century – in the year MinGuo 26 (1937) - Wang Chen-To proposed a realization of the specification from emperor Tsung and built a model from it. In 1947 George Lanchester successfully built a working model based on the principle of the differential gear.

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Since 1981 a South Pointing Chariot, built by professor Li Tian Yi's group at the Center University and based on George Lanchester reconstruction, is officially demonstrated in the National Science and Education Museum under topic 'Chinese Science and Technology History'.

At the National Sun Yat-Sen University a group of students from the department of mechanical engineering took part in a mechanism design project and attempted to rebuild imaginary South Pointing Chariots with traditional knowledge. They also got a lot of practical experience in going through all the phases from concept, design, construction, redesign, up to model building. During this process they have met a lot of obstacles and problems and needed many redesigns while finding new solutions.

2. Design Theory

From the specification handed down from emperor Tsung, we can find that the South Pointing Chariot uses a pin-gear drive train. George Lanchester explains the design as follows:

The South Pointing Chariot has two equal wheels. If it drives along a straight line, each wheel covers the same distance. If it circles along a fixed center point, the inside/outside wheels follow two circular curves with the same center point. The outside wheel has the longer curve. It is easily seen, that the turning angle has a direct proportion to the difference of the

two wheels' rotation. If the South Pointing Chariot does not drive along a circle, its way can be dissected into several small turns. Therefore, an angle is always directly proportional to the difference of the distance individually covered by the two wheels.
Using this differential value and turning back the pointing axle by an equal angle, the goal of pointing in a fixed direction is obtained.
Based on the tentative idea described above, we have created two concepts. One is the 'planetary gear system', the other 'screw-rod (dao-shue-gan)'. The design theories and plans are described below.

2-1: Planetary Gear System

For this model we have two different designs. As an introduction a short discussion of the basic construction of a planetary gear system will be given first (see figure 1).

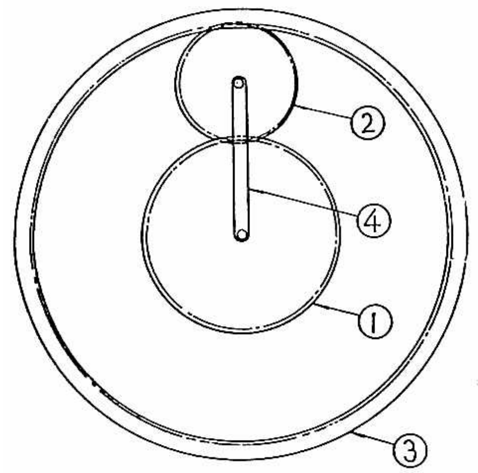

Figure 1: Planetary gear system

It consists of Sun Gear (1), Planet Gear (2), Ring Gear (3) and Planet Arm (4). With each part's radius (R_1 ... R_4) and Angular Velocity (W_1 ... W_4) we get:
Velocity of contact point A on the rim of the Sun Gear:

$$V_A = R_1 W_1$$

Velocity of outer end (Point B) of Planet Arm:

$$V_B = R_4 W_4 = (R_1 + R_2) W_4$$

Velocity of contact point A on the rim of the Planet Gear:

$$\underline{V}_A = \underline{V}_B + \underline{V}_{A/B} \quad (1)$$

Velocity of contact point C on the Ring Gear:

$$\underline{V}_C = \underline{V}_B + \underline{V}_{C/B} \quad (2)$$

As points A and C are rigidly coupled by the Planet Gear, it is assured that:

$$\underline{V}_{C/B} = -\underline{V}_{A/B} \quad (3)$$

Furthermore:

$$\underline{V}_{A/B} = \underline{V}_A - \underline{V}_B \quad (4)$$
$$\underline{V}_{C/B} = \underline{V}_C - \underline{V}_B \quad (5)$$

Combining equations (1) ... (4), (3) yields:

$$R_1 W_1 + R_3 W_3 = 2 (R_1 + R_2) W_4 \quad (6)$$

This can be used to determine the number of teeth (N) for each gear wheel:

(i) Immobilized Sun Gear ($W_1 = 0$):

$$SR = W_3 / W_4 = 2 (N_1 + N_2) / N_3 = 2 (R_1 + R_2) / R_3 \quad (7)$$

(ii) Immobilized Planet Arm ($W_4 = 0$):

$$SR = W_3 / W_1 = - N_1 / N_3 = - R_1 / R_3 \quad (8)$$

(iii) Immobilized Ring Gear ($W_3 = 0$):

$$SR = W_4 / W_1 = N_4 / [2 (N_1 + N_2)] = R_1 / [2 (R_1 + R_2)] \quad (9)$$

Based on this equations two designs are given.

Design A (see figure 2)

Gear 1 is fixed to the right road wheel.
Gear 8 is fixed to the left road wheel.
Gears 3 and 4 are step-up gears.
Gear 5 is the Sun Gear.
Gear 8 is the Ring Gear
Gears 6 and 7 are the Planet Gears

The Planet Arm function is taken by Gear 9, which turns Gear 10. The sense of rotation is restored by gears 12 and 13, so that the South Pointing function is achieved.

CASE 1: $W_{left} = W_{right}$, d.h. Gear 10 must not move at all. The Planet Gears must not move either => $W_9 = 0$

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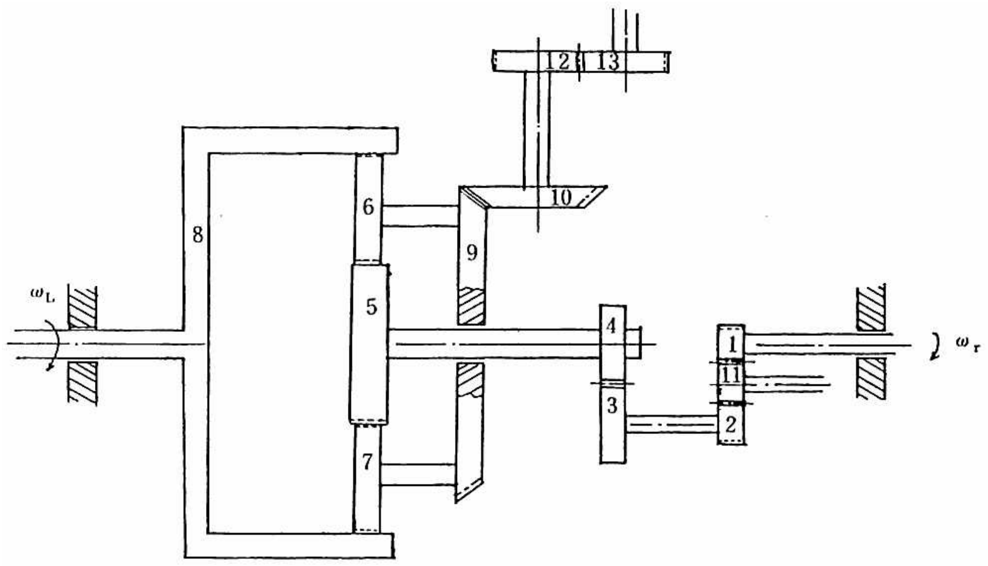

Figure 2: South Pointing Chariot mechanism design 1

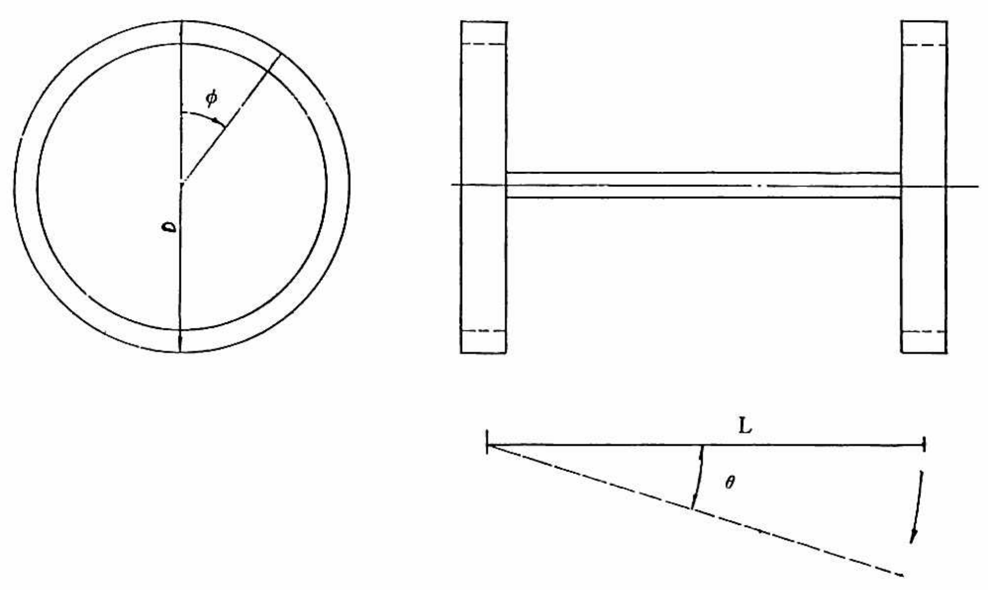

Figure 3: South Pointing Chariot body rotation

$W_8 = W_{left}$
$W_5 = W_{right} R_3/R_4$
$W_8 /W_5 = R_4/R_5$

Gears 4 and 5 are chosen so that

$$W_8 /W_5 = R_5/R_8 = R_4/R_5 \qquad (10)$$

CASE 2: The chariot turns left (see figure 3)
Assumed the right road wheel turns by an angle of W_Φ and $W_{left} = 0$, the Ring Gear doesn't turn either ($W_8 = 0$) and we get:

$$W_1 = W_\Phi = 2 L W_\Phi/D$$

This is scaled by Gears 3 and 4 to

$$W_5 = W_\Phi R_3 / R_4$$

It is known that:

$$W_9 / W_5 = W_5 / [2 (R_5 + R_6)] = R_5 / (2 R_9)$$

$$=> \quad W_9 = R_5 W_5 / (2 R_9)$$

What we need for South Pointing is a compensation angle $W_{10} = W_\Phi$

$$R_9 W_9 = R_{10} W_{10}$$

$$\Rightarrow R_{10} = R_5 W_5 / (2 R_9)$$
$$= (R_5/2) W_5 [2 L / (W_\Phi D)]$$
$$= (R_5 W_5 L) / (W_\Phi D)$$
$$= [(R_5 W_5 L) / D] [R_3 / (R_4 W_5)]$$
$$= (R_3 R_5 L) / (R_4 D) \qquad (11)$$

CASE 3: The chariot turns right
Assumed the left road wheel turns by an angle W_Φ, the right road wheel doesn't turn (W_{right} = 0) and consequenty the Sun Gear does not turn either ($W_5 = 0$).

$$\Rightarrow W_8 / W_9 = 2 (R_5 + R_6) / R_8 = 2 R_9 / R_8$$
$$W_9 = W_8 R_8 / (2 R_9)$$

Again we want a compensation angle $W_{10} = W_\Phi$

$$W_8 = W_\Phi$$
$$R_9 W_9 = R_9 (W_8 R_8) / (2 R_9) = R_{10} W_\Phi$$

It again holds valid that:

$$(D / 2) W_8 = L W_\Phi$$

$$R_{10} = R_8 W_8 / (2 W_\Phi)$$
$$= (R_8/2) [(2 / D) L W_\Phi] / W_\Phi$$
$$= (R_8 L) / D$$
$$= (R_3 R_5 L) / (R_4 D) \qquad (12)$$

Combining the equations (10) .. (12) from the three cases we get the solutions:

$$R_5 / R_8 = R_4 / R_3$$
$$R_{10} = (R_3 R_5 L) / (R_4 D)$$

And equivalently for the number of teeth

$$N_5 / N_8 = N_4 / N_3$$
$$N_{10} = (N_3 N_5 L) / (N_4 D)$$

Design B (see figure 4)
Gear 1 is the Sun Gear.
Gear 2 is the Planet Gear.
Gear 3 is the Ring Gear.
Reference 4 is the Planet Arm.
Bevel Gear 5 is driven by the right Road Wheel.
Gear 6 is driven by the left Road Wheel.
$N_1..N_6$ are the numbers of teeth,
$W_1..W_6$ the angular velocities,
$R_1.. R_6$ the radius and L the wheel base.

CASE 1: The chariot moves straight forward.
To keep the pointer steady we require:
$$W_4 = 0.$$
Obviously this is only possible if we have:
$$W_3 / W_1 = -N_1 / N_3$$
The construction defines
$$W_3 / W_6 = -N_6 / N_3$$
$$W_1 / W_5 = N_5 / N_1$$
$$(W_3 W_5) / (W_6 W_1) = -(R_6 R_1) / (R_3 R_5) = -(N_6 N_1) / (N_3 N_5)$$
Going straight implies
$$W_5 = W_6$$
If we postulate that
$$N_5 = N_1$$
The equation above collapses to
$$W_3 / W_1 = - N_6 / N_3$$
Comparing this to condition
$$W_3 / W_1 = -N_1 / N_3$$
From above yields
$$N_6 = N_1$$

CASE 2: The chariot turns right.
In the most simple form this is equivalent to:
$$W_6 \neq 0, W_5 = W_1 = 0$$
As proven above for the general planetary gear we get:
$$W_3 / W_4 = 2 (N_1 + N_2) / N_3$$

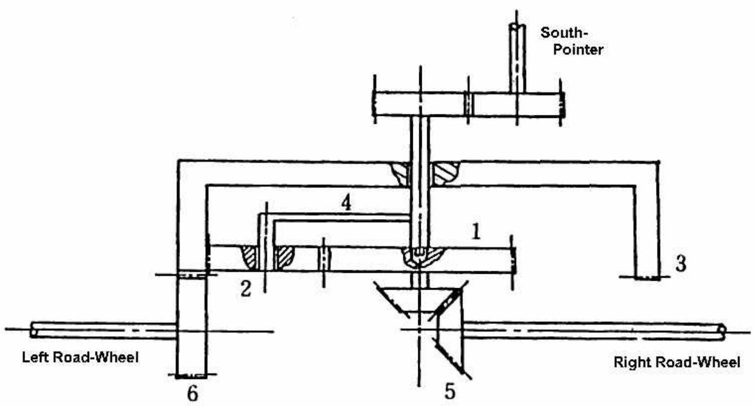

Figure 4: South Pointing Chariot mechanism design 2

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$W_3 / W_6 = - N_6 / N_3$
$\Rightarrow \quad W_3 = -(N_6 / N_3) W_6$

Combining the last two equations gives:

$W_4 = \{N_3 / [2 (N_1 + N_2)]\}\{ - N_6 / N_3\} W_6$
$= -\{N_6 / [2 (N_1 + N_2)]\} W_6$

Or written as pure angles (multiplied by time):

$$\Theta_4 = -\{N_6 / [2 (N_1 + N_2)]\} \Theta_6$$

CASE 3: The chariot turns left.
In the most simple form this is equivalent to:
$W_5 \neq 0, W_6 = W_3 = 0$
As proven above for the general planetary gear we get:

$W_4 / W_1 = N_3 / [2 (N_1 + N_2)]$

$W_1 / W_5 = N_5 / N_1$
$\Rightarrow W_1 = (N_5 / N_1) W_5$

Combining the last two equations gives:

$W_4 = \{N_1 / [2 (N_1 + N_2)]\}\{N_5 / N_1\} W_5$

Or written as pure angles (multiplied by time):

$$\Theta_4 = -\{N_5 / [2 (N_1 + N_2)]\} \Theta_5$$

Southpointing requires, that Planet Arm 4 turns by an complementary angle Θ_4 when the chariot turns by an angle Φ:

Right turn: $L \Phi = R \Theta_6$
From the requirement $\Phi = \Theta_4$ we get:

$R = | -N_6 / [2 (N_1 + N_2)] | L$
$= (N_6 L) / [2 (N_1 + N_2)]$

Left turn: $L \Phi = R \Theta_5$
From the requirement $\Phi = \Theta_4$ we get:

$R = (N_5 L) / [2 (N_1 + N_2)]$

R being the same for both road wheels requires
$N_5 = N_6$

If we combine this with the solution of case 1 we end up with:

$N_1 = N_5 = N_6$

Applying this to

$R = (N_6 L) / [2 (N_1 + N_2)]$

A valid example of such a chariot would be:

$N_1 = N_2 = N_5 = N_6 = 20$
$N_5 = 60$
$4 R = L$

2-2: Screw-rod system (Figure 5).

Ref. 1: Screw Rod
This rod is rigidly attached to the right road wheel, while the left road wheel uses it as an axle for free running.

Ref. 2: Nut
The nut is fixed to a disc (shown in the middle of figure 5). This disc has two holes with a sliding fit to two rods, which in turn are fixed to the left road wheel.

D: Road Wheel diameter
P: Pitch of the screw
L: Wheel base
Θ_r: Absolute angle the right road wheel is turned.
Θ_l: Absolute angle the left road wheel is turned.

Again we have to consider three cases:

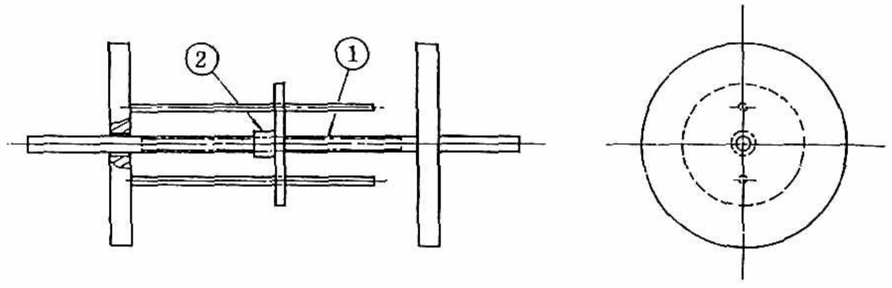

Figure 5: South Pointing Chariot mechanism design 3

CASE 1: $\Theta_r = \Theta_l$.
Both wheel turn with the same rate, nut (2) does not move relative to the rod (1).

CASE 2: $\Theta_r > \Theta_l$.
If the chariot sways left, the rod screws into the nut and thereby pulls the nut and its disc to the right by distance S depending on the angle Φ:

$$S = [(\Theta_r - \Theta_l) / (2\pi)]\, P \quad \text{travel to the right}$$

$$L\,\Phi = (D / 2)\,(\Theta_r - \Theta_l)$$
$$\Phi = [D / (2L)]\,(\Theta_r - \Theta_l) \quad \text{angle to the left}$$

CASE 3: $\Theta_r < \Theta_l$.
If the chariot sways right

$$S = [(\Theta_l - T_r) / (2\pi)]\, P \quad \text{travel to the left}$$

$$\Phi = [D / (2L)]\,(\Theta_l - T_r) \quad \text{angle to the right}$$

Cases 2 and 3 both reveal the same dependency between travel S of the nut (2) and the angle Φ the chariot turns:

$$S = [(P\,L) / (\pi D)]\,\Phi = C\,\Phi \qquad (13)$$

With constant C defined as

$$C = (P\,L) / (\pi D)$$

What we need next is a function f, which converts the linear translation (represented by S) into an proportional angle Φ:

$$f(S) = -\Phi$$

Mounted to the chariot, the output of f(S) will always point into the same direction – which is the principle of south-pointing. How can this be achieved? The most simple realization of function f is the relative movement between a rack and a pinion, the rack being shifted by nut (2). If the nut travels by a distance S, the rack is shifted by the same distance and turns the pinion by a certain angle.

Look at figure 5: When the chariot sways left by an angle F, the rack moves right by S and the pinion must turn left too. The requirement of function f is that the pinion turns exactly by angle F.
If this pinion is chosen with a diameter d we have:

$$S = (d / 2)\, f \qquad (14)$$

Merging equations (13) and (14) we get:

$$[(2\,P\,L) / (\pi D)]\,\Phi = (d / 2)\,\varphi$$

True southpointing requires:

$$\Phi = \varphi \Rightarrow d = (2\,P\,L) / (\pi D) \qquad (15)$$

So a pinion of diameter (2 P L) / (π D) is needed. The design is not possible in this form, as the pitch P of the nut (2) is usually very small and wheelbase L canot be chosen arbitrarily large. If some gear wheels with a total ratio K are used to enlarge the turning angle of the pinion, that is:

$$\varphi / K = \Phi / K$$

the pinion diameter is determined by:

$$d = (2\,P\,L\,K) / (\pi D) \qquad (16)$$

I other words, d can be chosen quite freely.

Figure 6 summarizes the explanation.

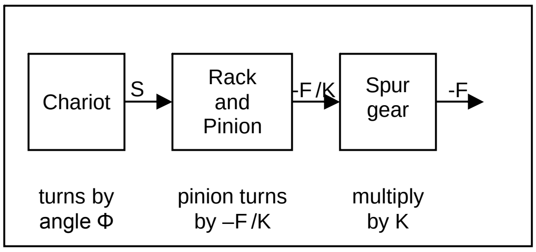

Figure 6: Block diagram of screw-rod SPC

3. Model Building and Summary

During the whole process of South Pointing Chariot building the problem was not the theory, but the requirement of manufacture. To tell the truth, the basic construction ideas and concepts were already closed within the first two to three months. The subsequent production process was delayed for several months. The seasons: Restricted production capability and limited funding. We got a budget of 3000 Taiwan dollars for each chariot. All materials were bought from an old ship on GuaoXong Avenue. Therefore, for the size of the design we always had to consider the existing parts, while manufacturing precision was constrained by handwork additionally.

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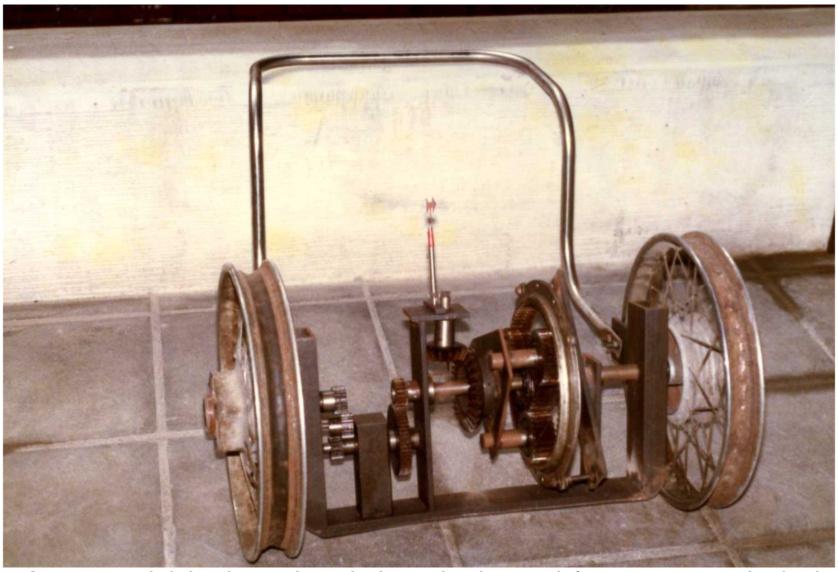

Figure 7: Finished South Pointing Chariot #1 (planetary gear design)

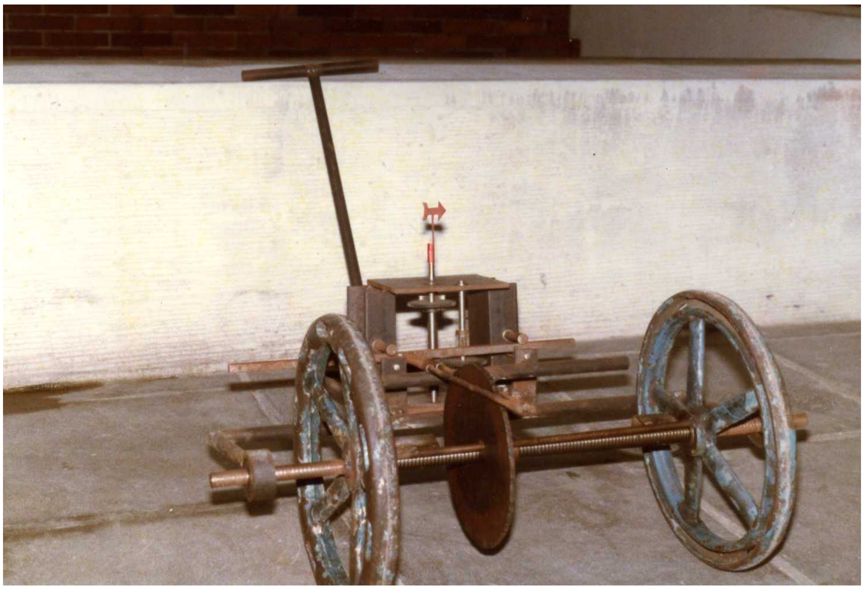

Figure 8: Finished South pointing Chariot #2 (screw-rod design)

So you probably can image the problems we had. In spite of all that we mastered the problems and trouble and two South Pointing Chariots were built in the end. See figures 7 and 8.

Because of the restrictions described above we have not spent a lot time for external finishing. The direction pointing precision is also limited by the production process and could not be achieved to be perfect in every way. But at least it was proven that our mechanism design concepts for a South Pointing Chariot are correct and realizable.

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